A Robustness Study of $\bar{X}$ Charts with Variable Sampling Intervals

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Recent theoretical studies have shown that, for all but very large process shifts, control charts using variable sampling interval (VSI) schemes are more efficient in their detection of shifting processes than the more conventional fixed sampling interval (FSI) schemes. This article, through simulation, considers the properties of the VSI $\bar{X}$ chart in an environment where the process data are not normally distributed but are contaminated. In addition, it evaluates the behavior of VSI charts where a trimmed mean, a winsorized mean, or the median is used as the chart statistic. Comparisons between the FSI and VSI $\bar{X}$ charts indicate that the VSI chart continues to be more efficient. Further, trimming the mean and/or including a rule for reducing the amount of switching between the two different sampling intervals improves the efficiency.

Introduction

IT is important in a commercial operating environment to detect an out-of-control process as soon as possible, while minimizing the chance that an in-control situation is interpreted as one requiring process change. The usual practice followed to accomplish this end is to sample the process on some fixed sampling interval (FSI) schedule, say hourly. As an alternative to this standard practice, Reynolds et al. (1988) proposed a variable sampling interval (VSI) scheme where the sampling interval, the time between samples, is not held fixed but is varied depending on the observed data. For example, if a sample point falls close to a control limit, the next sample is collected sooner, say in ten minutes. Conversely, if the sample falls close to the target, the next sample is taken after a longer period than the usual hour. Hence, with the VSI scheme the sampling interval will be shorter if there is some indication that there may be a problem and longer if there is no indication of a problem.

An example of a two-interval VSI chart is shown in Figure 1. The area between the 3-sigma Shewhart control limits is divided into the regions W_1, W_2, and C. The regions W_1 and W_2 are the warning regions and correspond to sampling time interval d_1, and C is the middle region corresponding to sampling time interval d_2 ($d_1 < d_2$). To implement the VSI scheme, one simply takes the next sample after a shorter time interval d_1 if the current control statistic falls in either region W_1 or W_2, and after a longer time interval d_2 if the control statistic falls in region C. Adding a 2/3 switching rule, one requires two of the last three control statistics to fall in region W_1 or two of the last three control statistics to fall in region W_2 before using sampling time interval d_1. Incorporating this switching rule reduces the average number of switches between the two sampling intervals.

Reynolds et al. (1988, 1990) presented theoretical and numerical comparisons based on normally distributed data between the FSI and VSI procedures. Their results show that the VSI chart offers a substantial benefit. Out-of-control situations are identified far more quickly for a fixed false alarm rate. The average run length (ARL), the average number of samples taken before the chart signals, is the same for both the VSI and FSI charts. However, the average time to signal (ATS), the expected time until the chart signals, can be made quite different when the sampling time interval is changed.

This paper investigates the performance of VSI charts in the presence of non-normal process distri-

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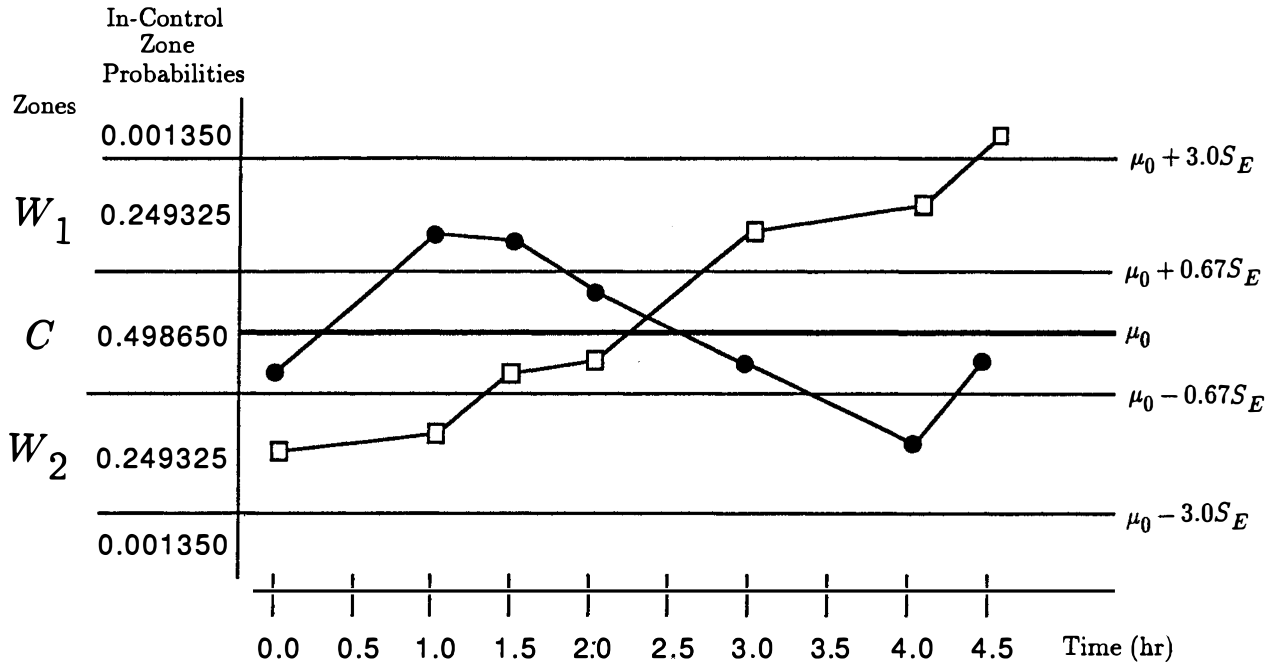

FIGURE 1. A VSI $\bar{X}$ Chart with Sampling Time Intervals $d_1 = 0.5$ and $d_2 = 1$. The Sequence of Means Denoted by Dots Portrays the VSI Scheme. The Sequence of Means Denoted by Squares Portrays the VSI Scheme with a 2/3 Switching Rule.

butions. Since standard control schemes are known to be influenced by nonnormal process distributions (e.g., see Lucas and Crosier (1982) and Rocke (1989)), the purpose of this paper is to evaluate the performance of the VSI $\bar{X}$ chart for a variety of contaminated normal distributions. In addition, charts are considered where the chart statistic is the 20% trimmed mean, the 20% winsorized mean, or the median. For simplicity the 20% trimmed mean and the 20% winsorized mean will just be referred to as the trimmed mean and the winsorized mean, respectively.

Literature Review

A brief review of the literature on sampling plans with more than one level of sampling is helpful in motivating the use of variable sampling interval process control charts. A continuous sampling plan (CSP-1) that alternates between 100% inspection and partial inspection was proposed by Dodge (1943). Other sampling plans followed, such as the multi-level sampling inspection plans of Lieberman and Solomon (1955) in which inspection intensity is increased when the manufacturing process is not operating well. Dodge (1955) applied the principles of continuous sampling to individual lots received from a production line. His skip-lot sampling plan (SkSP-1) makes provisions for "skipping" inspection of some fraction of the submitted lots when the quality of the inspected product is good.

The ideas of switching inspection levels in acceptance sampling carry over to statistical control charts

TABLE 1. Estimated Standard Errors of the Different Statistics

Population	Mean	Trimmed	Median	Winsorized
$N(0, 1)$ (Uncontaminated)	0.44721	0.47703	0.53389	0.47629
1% $N(0, 2.25)$	0.45000	0.47822	0.53813	0.47671
1% $N(0, 9.00)$	0.46476	0.48077	0.53878	0.47978
5% $N(0, 2.25)$	0.46098	0.48655	0.54649	0.48536
5% $N(0, 9.00)$	0.52915	0.50444	0.55883	0.50513
10% $N(0, 2.25)$	0.47434	0.49547	0.55494	0.49467
10% $N(0, 9.00)$	0.60000	0.53819	0.58861	0.54074
20% $N(0, 2.25)$	0.50000	0.51885	0.57948	0.51829
20% $N(0, 9.00)$	0.72111	0.61365	0.65143	0.62123

TABLE 2. AATS Values for the Populations in Table 1 (Assuming Normality)

			Displacement of Process Mean (Multiples of Standard Errors)						
Population	Statistic	Chart	0.0	0.5	1.0	1.5	2.0	3.0	4.0
$N(0, 1)$	Mean	FSI	369.00	153.71	43.08	14.37	5.83	1.50	0.68
		VSI	369.45	140.52	30.65	7.36	2.43	1.04	0.92
	Trimmed	FSI	367.46	156.64	43.20	14.38	5.72	1.52	0.68
		VSI	368.27	143.43	30.68	7.31	2.42	1.05	0.92
	Median	FSI	338.90	146.72	41.87	14.64	5.69	1.52	0.69
		VSI	339.56	134.43	29.68	7.46	2.42	1.05	0.92
	Windsorized	FSI	297.20	131.50	38.82	13.88	5.59	1.51	0.69
		VSI	293.23	119.34	27.52	7.15	2.43	1.05	0.93
1% $N(0, 2.25)$	Mean	FSI	364.49	156.32	43.14	14.80	5.94	1.48	0.68
		VSI	365.05	143.08	30.65	7.57	2.48	1.04	0.93
	Trimmed	FSI	364.68	148.32	43.81	14.33	5.85	1.50	0.68
		VSI	365.53	135.61	31.11	7.31	2.47	1.05	0.93
	Median	FSI	345.39	150.43	43.03	14.03	5.82	1.49	0.69
		VSI	347.14	137.71	30.55	7.22	2.44	1.04	0.93
	Windsorized	FSI	279.30	128.01	38.74	13.64	5.61	1.54	0.69
		VSI	274.91	115.72	27.67	7.04	2.38	1.05	0.93
5% $N(0, 2.25)$	Mean	FSI	337.83	148.60	42.93	14.49	5.83	1.52	0.69
		VSI	339.18	136.27	30.54	7.38	2.44	1.04	0.92
	Trimmed	FSI	345.71	152.42	43.92	14.60	5.94	1.50	0.69
		VSI	346.92	139.14	31.17	7.44	2.49	1.04	0.93
	Median	FSI	324.01	147.52	43.03	14.61	5.88	1.47	0.67
		VSI	326.55	135.25	30.50	7.47	2.45	1.03	0.92
	Windsorized	FSI	269.36	128.87	40.56	13.50	5.68	1.49	0.70
		VSI	265.61	116.71	28.83	7.03	2.46	1.04	0.93
10% $N(0, 2.25)$	Mean	FSI	321.19	143.20	41.88	14.32	6.07	1.52	0.68
		VSI	323.00	131.25	29.60	7.27	2.48	1.04	0.93
	Trimmed	FSI	319.52	138.95	42.00	14.37	5.93	1.48	0.69
		VSI	320.59	127.30	29.98	7.39	2.47	1.03	0.92
	Median	FSI	309.69	140.02	41.79	14.36	5.75	1.44	0.69
		VSI	311.16	128.18	29.67	7.32	2.39	1.04	0.93
	Windsorized	FSI	260.38	122.28	39.32	13.46	5.52	1.54	0.69
		VSI	256.78	110.81	27.84	7.03	2.38	1.05	0.93
20% $N(0, 2.25)$	Mean	FSI	295.38	139.99	41.90	14.54	5.85	1.53	0.69
		VSI	297.79	128.48	29.67	7.30	2.41	1.05	0.92
	Trimmed	FSI	307.97	140.55	42.37	14.60	5.94	1.46	0.69
		VSI	310.48	129.23	30.04	7.37	2.45	1.03	0.92
	Median	FSI	293.53	134.98	42.44	14.85	5.97	1.49	0.68
		VSI	296.99	124.27	30.05	7.48	2.45	1.04	0.92
	Windsorized	FSI	252.98	122.27	38.98	13.47	5.65	1.50	0.70
		VSI	251.48	111.29	27.63	6.95	2.42	1.05	0.93
1% $N(0, 9.00)$	Mean	FSI	229.39	126.42	42.46	14.72	5.96	1.50	0.68
		VSI	233.32	116.86	30.08	7.33	2.46	1.04	0.92
	Trimmed	FSI	328.23	142.47	42.87	14.48	5.80	1.49	0.68
		VSI	328.90	130.28	30.34	7.36	2.46	1.04	0.93
	Median	FSI	321.58	138.58	42.28	14.33	5.86	1.49	0.69
		VSI	322.66	126.52	30.05	7.26	2.46	1.04	0.92
	Windsorized	FSI	271.63	126.11	38.65	13.50	5.62	1.46	0.70
		VSI	268.08	113.78	27.53	7.09	2.43	1.04	0.93

TABLE 2—*Continued*

Population	Statistic	Chart	Displacement of Process Mean (Multiples of Standard Errors) 0.0	0.5	1.0	1.5	2.0	3.0	4.0
5% $N(0, 9.00)$	Mean	FSI	129.84	89.45	39.75	15.84	6.44	1.47	0.67
		VSI	137.78	84.78	27.83	7.53	2.46	1.04	0.93
	Trimmed	FSI	258.01	131.24	43.58	14.69	5.85	1.50	0.69
		VSI	261.45	121.29	30.79	7.36	2.39	1.04	0.93
	Median	FSI	262.14	133.94	41.97	14.66	5.82	1.50	0.68
		VSI	265.04	123.19	29.63	7.39	2.39	1.05	0.93
	Windsorized	FSI	222.16	116.40	39.00	13.95	5.67	1.48	0.69
		VSI	221.82	105.97	27.63	7.13	2.39	1.05	0.93
10% $N(0, 9.00)$	Mean	FSI	113.40	79.65	37.16	15.55	6.43	1.49	0.66
		VSI	123.00	76.68	25.83	7.19	2.42	1.06	0.93
	Trimmed	FSI	183.92	114.40	42.07	14.97	5.96	1.52	0.68
		VSI	189.29	106.95	29.70	7.39	2.44	1.04	0.93
	Median	FSI	217.83	123.72	42.94	15.08	6.11	1.50	0.71
		VSI	223.03	114.87	30.22	7.48	2.47	1.05	0.93
	Windsorized	FSI	168.93	101.19	39.10	14.39	5.93	1.52	0.67
		VSI	172.45	93.64	27.74	7.21	2.45	1.05	0.92
20% $N(0, 9.00)$	Mean	FSI	132.69	83.49	36.29	15.18	6.60	1.45	0.67
		VSI	144.38	80.14	24.97	6.99	2.52	1.05	0.93
	Trimmed	FSI	123.16	85.07	38.24	15.38	6.39	1.49	0.68
		VSI	130.26	80.89	26.79	7.31	2.47	1.05	0.93
	Median	FSI	150.24	100.51	41.61	15.30	6.31	1.52	0.68
		VSI	157.05	94.58	29.29	7.42	2.46	1.04	0.92
	Windsorized	FSI	113.80	78.98	37.42	15.07	6.23	1.48	0.67
		VSI	120.19	74.80	26.24	7.21	2.44	1.04	0.93

for monitoring the output of a production process. Arnold (1970) proposed sampling procedures that use variable sampling intervals, but he did not consider the problem of controlling a process by using a control chart. Sampling schemes that vary the time between samples were also investigated by Smeach and Jernigan (1977), and Crigler and Arnold (1979, 1986). More recently, Reynolds et al. (1988, 1990), Amin and Letsinger (1991), and others investigated the properties of various process control procedures when variable sampling intervals are used. All previous work on VSI process control procedures assumed the normal distribution as the underlying distribution of the chart statistic.

Properties of the VSI Chart

The properties of a FSI control procedure often are determined by the ARL. The ARL should be long when the process is operating on target so that the false alarm rate is low, and the ARL should be short when the process means shifts. With a VSI control procedure it is also necessary to consider the ATS. Reynolds et al. (1988) pointed out that in the out-of-control situation it is more realistic to consider the adjusted average time to signal (AATS), which allows for the shift to occur *between* two samples. They developed a model for the time of the shift that assumes the position of the shift is uniformly distributed over the interval.

Another feature of VSI control procedures is the switching between the short and long sampling interval. Excessive switching can be a complicating factor in the application of VSI procedures, especially when the process is in control. Amin and Letsinger (1991) proposed the average number of switches (ANSW) as a useful criterion for evaluating the switching behavior of VSI control procedures. The ANSW is defined as the expected number of switches from the start of the process until the chart signals. Amin and Hemasinha (1991) provide analytical results on the switching behavior of $\bar{X}$ charts that assist in the design of VSI charts with runs rules.

When comparing several VSI control procedures, the chart parameters values are chosen to give the same in-control ARL and ATS values. The control procedure that has the lowest out-of-control AATS value at a specific shift is considered most efficient for that shift.

The Simulations

For each simulation the AATS, the ARL, the ANSW, and the average number of times the statistic was plotted in zones W_1, W_2, and C were recorded. When comparing VSI and FSI charts, the two sampling time intervals were chosen as $d_1 = 0.1$ and $d_2 = 1.9$ so that the in-control expected VSI sampling time interval was equal to the fixed sampling time interval ($d = 1$). For these simulations, five observations were used to calculate each statistic (mean, trimmed mean, winsorized mean, and median). The sample standard errors ($S'_E s$) presented in Table 1 were calculated from 50,000 simulated values of each statistic. In Case I the contamination was treated as unsuspected. The data were treated as if they were normally distributed, and chart control limits were determined for each contaminated population using the values in Table 1. Outer control limits were set at $\pm 3S_E$, inner control limits were set at $\pm 0.6723 S_E$, and the target value $\mu_0 = 0$ was used as the center line. In practice the grand mean $\bar{\bar{X}}$ is often used as the center line of an $\bar{X}$ chart.

In Case II, normality was not assumed. Outer control limits were set by sampling the distribution 50,000 times (each sample being a subgroup of size five) and setting control limits such that a specified proportion of the observations fell within the control limits. Inner limits were set at the inner quartiles (i.e., such that 25% of the samples were above the upper limit and 25% were below the lower limit).

Contaminated Normal Populations

For Case I, where control limits were set based on the assumption that the contaminated population was normally distributed (an assumption that was invalid but likely to be made when the population is characterized by too few samples or the contamination is not extreme), two things happened. First, the in-control ATS was reduced from the expected 370, and second, both the VSI and FSI charts lost sensitivity to process shifts. However, except in some cases when using a winsorized mean, the in-control ATS was reduced less for the VSI charts, and in every case the VSI chart responded faster to process shifts than its FSI counterpart. For the eight contaminated distributions shown in Table 2, the VSI mean chart responded on the average 12.04 time units faster to a one sigma shift in the process than the corresponding FSI charts. This behavior compares well to the 12.43 time unit differential observed for the uncontaminated distribution.

For further comparison in the situation where the in-control ATS was changing, the AATS was normalized by taking the ratio of the AATS value to the in-control ATS value to obtain

$$\text{AATS}^*(\mu_1) = \text{AATS}(\mu_1)/\text{ATS}(\mu_0)$$

where μ_0 is the mean of the process when it is in control. While AATS values are dependent on both the level of contamination and the magnitude of the process shift, AATS* values are more influenced by the latter. Hence, this ratio provides a direct measure of the speed with which a chart responds to a shift. Table 3 gives AATS* values for selected distributions. The smaller the AATS* value, the faster the statistic's response to a one sigma shift. Again, the VSI chart was the more responsive in each case. Deviations from normality through contamination led both to an increased tendency to adjust the process when it was not needed (i.e., the in-control ATS dropped) and to not adjust the process when it was needed (i.e., the rate of response slowed).

In Case II, where no assumption concerning normality was made, the situation was similar. Tables 3 and 4 show AATS* and AATS values, respectively. While the VSI was again better than the FSI in every case, for the distribution with slight contamination (1% $N(0,9)$) the AATS* values for the mean statistic were similar to their Case I counterparts, but at high contamination (20% $N(0,9)$) the AATS* values improved (i.e., decreased) when compared with their Case I counterparts. Further, the robust measures were not as sensitive as the mean when the process shifted and the contamination was at a high level (20%). By not assuming normality, the in-control AATS was less affected by contamination, but the sensitivity to process shift was reduced significantly.

Robust Measures

Lucas (1982) showed that the trimmed mean is efficient for control when the distribution is slightly long-tailed, whereas, a winsorized mean is efficient for a short tailed distribution. These robust control statistics were evaluated for the various FSI and VSI procedures (Tables 2, 3, and 4). By definition, an efficient sampling plan should have a low false alarm rate when no process shift has occurred, yet respond quickly when the process does shift. For heavy contamination levels, the median met this criteria best, while for light and moderate levels the

TABLE 3. AATS Values at $\mu_1 = \mu_0 + S_E$

Population	Mean FSI	Mean VSI	Trimmed FSI	Trimmed VSI	Median FSI	Median VSI	Winsorized FSI	Winsorized VSI
				Assume Normality (Case I)				
100% $N(0, 1)$	0.117	0.083	0.118	0.083	0.124	0.087	0.131	0.094
1% $N(0, 2.25)$	0.118	0.084	0.120	0.085	0.125	0.088	0.139	0.101
5% $N(0, 2.25)$	0.127	0.090	0.127	0.090	0.133	0.093	0.151	0.109
10% $N(0, 2.25)$	0.130	0.092	0.131	0.094	0.135	0.095	0.151	0.108
20% $N(0, 2.25)$	0.142	0.100	0.138	0.097	0.145	0.101	0.154	0.110
1% $N(0, 9.00)$	0.185	0.129	0.131	0.092	0.131	0.093	0.142	0.103
5% $N(0, 9.00)$	0.306	0.202	0.169	0.118	0.160	0.112	0.178	0.125
10% $N(0, 9.00)$	0.328	0.210	0.229	0.157	0.197	0.135	0.231	0.161
20% $N(0, 9.00)$	0.273	0.173	0.310	0.206	0.277	0.187	0.329	0.218
				No Normality Assumption (Case II)				
1% $N(0, 9.00)$	0.186	0.130	0.145	0.104	0.137	0.099	0.146	0.104
20% $N(0, 9.00)$	0.251	0.154	0.349	0.225	0.298	0.196	0.379	0.242

trimmed mean was to be preferred. With increasing contamination levels, the simple (arithmetic) mean led to many false signals when the process had not shifted, and at the same time it was not responsive enough when the process did shift. Further, the trimmed mean responded much like the mean when no contamination was present. That is, the distribution of the trimmed samples taken from an uncontaminated normal population appeared to be normal when the 50,000 member sample was tested using Kolomogorov's goodness-of-fit test (see, e.g., Conover (1980)). The ARL response curves of the trimmed mean chart are similar to those of the mean chart, and relative to the mean chart, the variance of the trimmed mean is just slightly inflated.

In Case II, behavior of the simple mean was better at high contamination levels, but the robust measures were considerably more sensitive at low contamination levels. Relative to the Case I counterpart, the

TABLE 4. AATS* Values for Two Contaminated Populations (Assuming Normality)

Population	Statistic	Chart	ATS (μ_0)	Displacement of Process Mean (Multiples of Standard Errors) 0.5	1.0	1.5	2.0	3.0	4.0
1% $N(0, 9.0)$	Mean	FSI	250	138.05	46.46	16.07	6.26	1.58	0.70
		VSI	250	126.94	32.61	7.92	2.50	1.05	0.93
	Trimmed	FSI	250	116.78	36.37	12.24	5.12	1.36	0.66
		VSI	250	107.69	26.11	6.39	2.24	1.03	0.92
	Median	FSI	250	112.54	34.28	12.23	5.05	1.42	0.66
		VSI	250	103.33	24.65	6.42	2.26	1.03	0.92
	Windsorized	FSI	250	117.70	36.61	13.26	5.49	1.46	0.68
		VSI	250	109.96	26.09	6.98	2.39	1.05	0.93
20% $N(0, 9.0)$	Mean	FSI	250	155.57	62.84	25.67	10.91	2.27	0.79
		VSI	250	136.31	38.61	10.25	3.33	1.15	0.94
	Trimmed	FSI	250	178.68	87.19	34.16	13.58	2.58	0.86
		VSI	250	159.78	56.17	14.27	4.00	1.17	0.94
	Median	FSI	250	172.72	74.38	28.19	10.57	2.14	0.80
		VSI	250	154.73	49.10	12.22	3.38	1.11	0.94
	Windsorized	FSI	250	180.88	94.86	38.77	15.31	2.92	0.93
		VSI	250	159.79	60.58	15.78	4.40	1.22	0.95

in-control ATS was longer, hence better, but the shift sensitivity suffered.

A 2/3 Switching Rule

An "ideal" VSI control scheme uses the long sampling time interval (d_2) exclusively when the process is operating in control, and it uses the short sampling time interval (d_1) exclusively when the process has changed. Amin and Letsinger (1991) and Amin and Hemasinha (1991) demonstrated that a 2/3 switching rule improves the behavior of the VSI chart for normally distributed data. In this study it is shown that the VSI chart with a 2/3 switching rule is more efficient, requiring fewer switches between sampling time intervals and using fewer short intervals when the process is in-control.

The improvement gained by using the 2/3 switching rule and the trimmed mean is demonstrated in Tables 5 and 6. The design parameters of the VSI $\bar{X}$ charts were chosen such that the in-control ARL and ATS values were approximately the same for the charts with the 2/3 switching rule and for the corresponding charts without the 2/3 switching rule. In order to ensure a fair comparison, we also selected the parameters such that all charts used the same sampling time intervals and had the same corresponding probabilities. For example, the following two VSI $\bar{X}$ charts have approximately the same in-control ARL, ATS, and ANSW values for $d_1 = 0.10$ and $d_2 = 1.90$.

Chart a: with no switching rule

$$W_2 = (-3S_E, -0.6723S_E]$$
$$C = (-0.6723S_E, 0.6723S_E)$$
$$W_1 = [0.6723S_E, 3S_E)$$

Chart b: with a 2/3 switching rule

$$W_2 = (-3S_E, -0.4496S_E]$$
$$C = (-0.4496S_E, 0.4496S_E)$$
$$W_1 = [0.4496S_E, 3S_E)$$

Amin and Hemasinha (1991) provide the necessary results for selecting design parameters.

The use of the 2/3 switching rule made the VSI $\bar{X}$ chart more sensitive to shifts in the process mean as evidenced by the smaller AATS* values in Table 5. Table 6 gives the average number of switches in addition to the number of times the sampling interval d_1 is used. Obviously, the chart with the 2/3 switching rule has a considerably smaller average number of switches across all shifts. This is true for both the arithmetic mean and the trimmed mean. For example, when the trimmed mean is used with the 2/3 switching rule, the in-control number of switches are only 106.5 (for an in-control ATS of 362.2). Without the use of the 2/3 switching rule, the number of switches are 185.9 for an in-control ATS of 368.3.

When a sample mean falls in the warning region (W_1 or W_2), this is an indication that the process mean may have shifted. The use of d_1 when the process is in control can be viewed as a "false alarm" since frequent sampling is an undesirable feature when the process is in control. On the other hand, early detection of a process shift would require that d_1 be used more frequently than d_2 when the process is out of control. Tables 5 and 6 show that the use of the 2/3 switching rule considerably reduced the ANSW without a loss of power. At the

TABLE 5. AATS* Values Using a 2/3 Switching Rule (Assuming Normality)

				Adjusted Average Time to Signal					
				Displacement of Process Mean (Multiples of Standard Errors)					
Population	Chart Statistic		ATS (μ_0)	0.5	1.0	1.5	2.0	3.0	4.0
$N(0, 1)$	Mean	FSI	369.0	0.417	0.117	0.039	0.016	0.004	0.002
	Mean	VSI	369.5	0.380	0.083	0.020	0.007	0.003	0.002
	Mean$_{2/3}$	VSI	374.0	0.344	0.056	0.013	0.006	0.004	0.002
	Trimmed	VSI	368.3	0.389	0.083	0.020	0.007	0.003	0.003
	Trimmed$_{2/3}$	VSI	362.2	0.349	0.059	0.013	0.006	0.003	0.002
1% $N(0, 9.00)$	Mean	FSI	229.4	0.551	0.185	0.064	0.026	0.007	0.003
	Mean	VSI	233.3	0.500	0.129	0.031	0.011	0.004	0.004
	Mean$_{2/3}$	VSI	231.4	0.447	0.091	0.020	0.010	0.005	0.004
	Trimmed	VSI	328.9	0.396	0.092	0.022	0.007	0.003	0.003
	Trimmed$_{2/3}$	VSI	338.7	0.360	0.062	0.014	0.007	0.004	0.002

TABLE 6. ANSW Values and the Average Number of Times the Sampling Time Interval d_1 is Used (Assuming Normality)

			Characteristics of VSI Charts						
			Displacement of Process Mean (Multiples of Standard Errors)						
Population	Chart Statistic		0.0	0.5	1.0	1.5	2.0	3.0	4.0
$N(0, 1)$	Mean	Num d_1	184.4	83.7	29.1	11.3	4.7	1.0	0.2
		ANSW	184.2	75.5	19.5	5.1	1.5	0.5	0.2
	$\text{Mean}_{2/3}$	Num d_1	185.4	95.2	33.6	12.2	4.8	0.8	0.1
		ANSW	110.1	42.1	7.7	1.5	0.7	0.3	0.1
	Trimmed	Num d_1	185.3	85.1	28.7	11.2	4.6	1.0	0.2
		ANSW	185.9	77.0	19.2	5.0	1.5	0.5	0.2
	$\text{Trimmed}_{2/3}$	Num d_1	179.2	92.4	34.4	12.9	4.8	0.8	0.1
		ANSW	106.5	41.2	7.8	1.6	0.7	0.3	0.1
1% $N(0, 9.00)$	Mean	Num d_1	113.2	68.8	28.8	11.6	5.0	1.0	0.2
		ANSW	115.1	62.7	19.2	5.1	1.6	0.5	0.1
	$\text{Mean}_{2/3}$	Num d_1	110.7	74.9	34.3	13.0	4.9	0.7	0.1
		ANSW	66.9	33.5	7.8	1.6	0.7	0.3	0.1
	Trimmed	Num d_1	164.1	80.8	27.9	11.0	4.8	1.0	0.2
		ANSW	164.4	73.0	18.9	4.9	1.6	0.5	0.1
	$\text{Trimmed}_{2/3}$	Num d_1	167.6	89.8	33.5	12.4	4.8	0.8	0.1
		ANSW	99.6	39.7	7.7	1.5	0.7	0.3	0.1

same time, the out-of-control AATS values are often smaller for the charts with the 2/3 switching rule than for the corresponding charts without the 2/3 switching rule. Similar improvements due to the use of the 2/3 switching rule were observed for both the arithmetic mean and the trimmed mean with a contaminated population. The reduction of the ANSW values is a significant improvement since having large numbers of switches may complicate the administration of VSI schemes.

Summary

In conclusion, the VSI charting approach, reported by Reynolds et al. (1988) to be useful for normally distributed data, continues to be efficient in detecting process shifts when the data are distributed as a contaminated normal. Generally, as the contamination level increased, either in percent contamination or in greater contaminate variance, the in-control ATS values for both the VSI and FSI schemes dropped, more so for the FSI. The VSI scheme was slower to signal a false process shift in the in-control situation and, at the same time, more sensitive to actual shifts.

The trimmed mean performed better than the mean when the normal distribution was contaminated and yet did not suffer significantly when the distribution was normal. When the contamination level was high the median performed best. In no case did the winsorized mean outperform the trimmed means. Adding a 2/3 switching rule did not result in a loss of sensitivity of the VSI chart to process shifts. The VSI chart incorporating the trimmed mean and a 2/3 switching rule seems to provide a very effective control scheme for a commercial operation where normal distributions are likely to be contaminated.

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